



**NAMIBIA UNIVERSITY  
OF SCIENCE AND TECHNOLOGY**

FACULTY OF HEALTH AND APPLIED SCIENCES  
DEPARTMENT OF MATHEMATICS

|   |   |
|---|---|
| <b>QUALIFICATION:</b> BACHELOR OF SCIENCE IN APPLIED MATHEMATICS AND STATISTICS |   |
| <b>PROGRAMME CODE:</b> 07BAMS   | <b>LEVEL:</b> 7                                       |
| <b>COURSE CODE:</b> DAE702S   | <b>COURSE NAME:</b> DESIGN AND ANALYSIS OF EXPERIMENT |
| <b>DATE:</b> Jan-2023   | <b>PAPER:</b> THEORY                                  |
| <b>DURATION:</b> 3 HOURS  | <b>MARKS:</b> 100                                     |

**SUPPLEMENTARY/ 2ND OPPORTUNITY EXAMINATION  
QUESTION PAPER**

|                  |                   |
|------------------|-------------------|
| <b>EXAMINER</b>  | Dr. Jacob Ong'ala |
| <b>MODERATOR</b> | Prof Peter Njuho  |

**INSTRUCTION**

1. Answer ALL the questions in the booklet provided
2. Scan your answer sheet and upload it in the e-learning
2. Show clearly all the steps used in the calculation
3. All written work must be shown in the answer sheet.

**PERMISSIBLE MATERIALS**

Non-programmable calculator without cover

**THIS QUESTION PAPER CONSISTS OF 15 PAGES** (including the front page and attachments)

**QUESTION ONE - 27 MARKS**

- (a) A computer ANOVA output is shown below. Fill in the blanks. (You may give bounds on the *P*-value) [5 mks]

| <b>One-way ANOVA</b> |    |         |        |   |   |
|----------------------|----|---------|--------|---|---|
| Source               | DF | SS      | MS     | F | P |
| Factor               | ?  | ?       | 246.93 | ? | ? |
| Error                | 25 | 186.53  | ?      |   |   |
| Total                | 29 | 1174.24 |        |   |   |

- (b) Several ovens in a metal working shop are used to heat metal specimens. All the ovens are supposed to operate at the same temperature, although it is suspected that this may not be true. Three ovens are selected at random, and their temperatures on successive heats are noted. The data collected are as follows:(Use  $\alpha = 0.05$ )

| Oven | Temperature |        |        |        |        |        |
|------|-------------|--------|--------|--------|--------|--------|
| 1    | 491.50      | 498.30 | 498.10 | 493.50 | 493.60 |        |
| 2    | 488.50      | 484.65 | 479.90 | 477.35 |        |        |
| 3    | 490.10      | 484.80 | 488.25 | 473.00 | 471.85 | 478.65 |

- (i) Construct ANOVA table [12 mks]  
 (ii) Formulate the hypothesis when random effect model is assumed [2 mks]  
 (iii) What do you conclude from the ANOVA table above? [3 mks]  
 (iv) Estimate variance component [5 mks]

**QUESTION TWO - 22 MARKS**

Suppose that four normal populations have means of  $\mu_1 = 50$ ,  $\mu_2 = 60$ ,  $\mu_3 = 50$ , and  $\mu_4 = 60$ .

- (a) How many observations should be taken from each population so that the probability of rejecting the null hypothesis of equal population means is at least 0.90? Assume that  $\alpha = 0.05$  and that a reasonable estimate of the error variance is  $\sigma^2 = 25$ . [10 mks]  
 (b) How would your answer change if a reasonable estimate of the experimental error variance were  $\sigma^2 = 36$  [5 mks]  
 (c) How would your answer change if a reasonable estimate of the experimental error variance were  $\sigma^2 = 49$  [5 mks]  
 (d) Can you draw any conclusions about the sensitivity of your answer in this particular situation about how your estimate of  $\sigma$  affects the decision about sample size? [2 mks]

**QUESTION THREE - 28 MARKS**

- (a) The ANOVA from a randomized complete block experiment output is shown below.

| Source    | DF | SS      | MS     | F     | P |
|-----------|----|---------|--------|-------|---|
| Treatment | 4  | 1010.56 | ?      | 29.84 | ? |
| Block     | ?  | ?       | 64.765 | ?     | ? |
| Error     | 20 | 169.33  | ?      |       |   |
| Total     | 29 | 1503.71 |        |       |   |

- (i) Fill in the blanks [4 mks]
- (ii) How many blocks were used in this experiment? [1 mks]
- (iii) What conclusions can you draw? [2 mks]
- (b) A chemist wishes to test the effect of four chemical agents on the strength of a particular type of cloth. Because there might be variability from one bolt to another, the chemist decides to use a randomized block design, with the bolts of cloth considered as blocks. She selects five bolts and applies all four chemicals in random order to each bolt. The resulting tensile strengths are as follow

| Chemical | Bolt |    |    |    |    |
|----------|------|----|----|----|----|
|          | 1    | 2  | 3  | 4  | 5  |
| 1        | 73   | 68 | 74 | 71 | 67 |
| 2        | 73   | 67 | 75 | 72 | 70 |
| 3        | 75   | 68 | 78 | 73 | 68 |
| 4        | 73   | 71 | 75 | 75 | 69 |

- (i) Analyze the data from this experiment. [12 mks]
- (ii) Use the Fisher LSD method to make comparisons among the four Chemicals [9 mks]

**QUESTION FOUR - 23 MARKS**

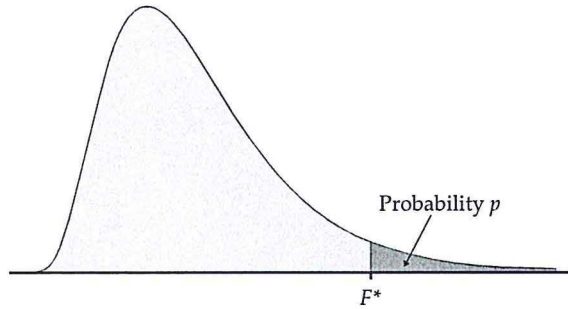
Suppose you want to determine whether the brand of laundry detergent used and the temperature affects the amount of dirt removed from your laundry. To this end, you buy two different brand of detergent ("Super" and "Best") and choose three different temperature levels ("cold", "warm", and "hot"). Then you divide your laundry randomly into  $6 \times r$  piles of equal size and assign each  $r=4$  piles into the combination of ("Super" and "Best") and ("cold", "warm", and "hot"). The amounts  $4Y_{ijk}$  of dirt removed when washing sub pile  $k(k = 1, 2, 3, 4)$  with detergent  $i(i = 1, 2)$  at temperature  $j(j = 1, 2, 3)$  are recorded in the table below.

|       | Cold    | Warm        | Hot         |
|-------|---------|-------------|-------------|
| Super | 4,5,6,5 | 7,9,8,12    | 10,12,11,9  |
| Best  | 6,6,4,4 | 13,15,12,12 | 12,13,10,13 |

- (a) Formulate the hypotheses for this experiment [2 mks]
- (b) Test the hypothesis above using Analysis of Variance [17 mks]
- (c) What do you conclude from the ANOVA results above [4 mks]

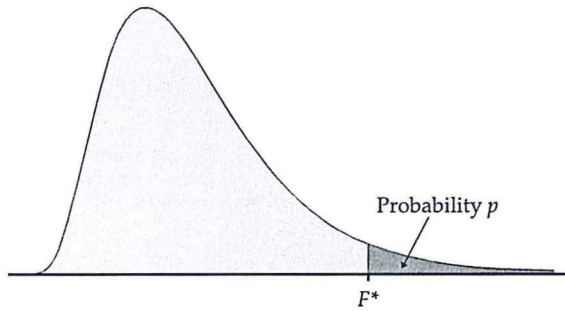
– END OF QUESTIONS –

Table entry for  $p$  is the critical value  $F^*$  with probability  $p$  lying to its right.



| TABLE E                               |      | F critical values                   |        |        |        |        |        |        |        |        |        |
|---------------------------------------|------|-------------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|                                       |      | Degrees of freedom in the numerator |        |        |        |        |        |        |        |        |        |
| $p$                                   |      | 1                                   | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      |        |
| Degrees of freedom in the denominator | 1    | .100                                | 39.86  | 49.50  | 53.59  | 55.83  | 57.24  | 58.20  | 58.91  | 59.44  | 59.86  |
|                                       |      | .050                                | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 |
|                                       |      | .025                                | 647.79 | 799.50 | 864.16 | 899.58 | 921.85 | 937.11 | 948.22 | 956.66 | 963.28 |
|                                       |      | .010                                | 4052.2 | 4999.5 | 5403.4 | 5624.6 | 5763.6 | 5859.0 | 5928.4 | 5981.1 | 6022.5 |
|                                       |      | .001                                | 405284 | 500000 | 540379 | 562500 | 576405 | 585937 | 592873 | 598144 | 602284 |
|                                       | 2    | .100                                | 8.53   | 9.00   | 9.16   | 9.24   | 9.29   | 9.33   | 9.35   | 9.37   | 9.38   |
|                                       |      | .050                                | 18.51  | 19.00  | 19.16  | 19.25  | 19.30  | 19.33  | 19.35  | 19.37  | 19.38  |
|                                       |      | .025                                | 38.51  | 39.00  | 39.17  | 39.25  | 39.30  | 39.33  | 39.36  | 39.37  | 39.39  |
|                                       |      | .010                                | 98.50  | 99.00  | 99.17  | 99.25  | 99.30  | 99.33  | 99.36  | 99.37  | 99.39  |
|                                       |      | .001                                | 998.50 | 999.00 | 999.17 | 999.25 | 999.30 | 999.33 | 999.36 | 999.37 | 999.39 |
|                                       | 3    | .100                                | 5.54   | 5.46   | 5.39   | 5.34   | 5.31   | 5.28   | 5.27   | 5.25   | 5.24   |
|                                       |      | .050                                | 10.13  | 9.55   | 9.28   | 9.12   | 9.01   | 8.94   | 8.89   | 8.85   | 8.81   |
|                                       |      | .025                                | 17.44  | 16.04  | 15.44  | 15.10  | 14.88  | 14.73  | 14.62  | 14.54  | 14.47  |
|                                       |      | .010                                | 34.12  | 30.82  | 29.46  | 28.71  | 28.24  | 27.91  | 27.67  | 27.49  | 27.35  |
|                                       |      | .001                                | 167.03 | 148.50 | 141.11 | 137.10 | 134.58 | 132.85 | 131.58 | 130.62 | 129.86 |
|                                       | 4    | .100                                | 4.54   | 4.32   | 4.19   | 4.11   | 4.05   | 4.01   | 3.98   | 3.95   | 3.94   |
|                                       |      | .050                                | 7.71   | 6.94   | 6.59   | 6.39   | 6.26   | 6.16   | 6.09   | 6.04   | 6.00   |
|                                       |      | .025                                | 12.22  | 10.65  | 9.98   | 9.60   | 9.36   | 9.20   | 9.07   | 8.98   | 8.90   |
|                                       |      | .010                                | 21.20  | 18.00  | 16.69  | 15.98  | 15.52  | 15.21  | 14.98  | 14.80  | 14.66  |
|                                       |      | .001                                | 74.14  | 61.25  | 56.18  | 53.44  | 51.71  | 50.53  | 49.66  | 49.00  | 48.47  |
| 5                                     | .100 | 4.06                                | 3.78   | 3.62   | 3.52   | 3.45   | 3.40   | 3.37   | 3.34   | 3.32   |        |
|                                       | .050 | 6.61                                | 5.79   | 5.41   | 5.19   | 5.05   | 4.95   | 4.88   | 4.82   | 4.77   |        |
|                                       | .025 | 10.01                               | 8.43   | 7.76   | 7.39   | 7.15   | 6.98   | 6.85   | 6.76   | 6.68   |        |
|                                       | .010 | 16.26                               | 13.27  | 12.06  | 11.39  | 10.97  | 10.67  | 10.46  | 10.29  | 10.16  |        |
|                                       | .001 | 47.18                               | 37.12  | 33.20  | 31.09  | 29.75  | 28.83  | 28.16  | 27.65  | 27.24  |        |
| 6                                     | .100 | 3.78                                | 3.46   | 3.29   | 3.18   | 3.11   | 3.05   | 3.01   | 2.98   | 2.96   |        |
|                                       | .050 | 5.99                                | 5.14   | 4.76   | 4.53   | 4.39   | 4.28   | 4.21   | 4.15   | 4.10   |        |
|                                       | .025 | 8.81                                | 7.26   | 6.60   | 6.23   | 5.99   | 5.82   | 5.70   | 5.60   | 5.52   |        |
|                                       | .010 | 13.75                               | 10.92  | 9.78   | 9.15   | 8.75   | 8.47   | 8.26   | 8.10   | 7.98   |        |
|                                       | .001 | 35.51                               | 27.00  | 23.70  | 21.92  | 20.80  | 20.03  | 19.46  | 19.03  | 18.69  |        |
| 7                                     | .100 | 3.59                                | 3.26   | 3.07   | 2.96   | 2.88   | 2.83   | 2.78   | 2.75   | 2.72   |        |
|                                       | .050 | 5.59                                | 4.74   | 4.35   | 4.12   | 3.97   | 3.87   | 3.79   | 3.73   | 3.68   |        |
|                                       | .025 | 8.07                                | 6.54   | 5.89   | 5.52   | 5.29   | 5.12   | 4.99   | 4.90   | 4.82   |        |
|                                       | .010 | 12.25                               | 9.55   | 8.45   | 7.85   | 7.46   | 7.19   | 6.99   | 6.84   | 6.72   |        |
|                                       | .001 | 29.25                               | 21.69  | 18.77  | 17.20  | 16.21  | 15.52  | 15.02  | 14.63  | 14.33  |        |

Table entry for  $p$  is the critical value  $F^*$  with probability  $p$  lying to its right.



| TABLE E                             |        |        |        |        |        |        |        |        |        |        |
|-------------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| F critical values (continued)       |        |        |        |        |        |        |        |        |        |        |
| Degrees of freedom in the numerator |        |        |        |        |        |        |        |        |        |        |
| 10                                  | 12     | 15     | 20     | 25     | 30     | 40     | 50     | 60     | 120    | 1000   |
| 60.19                               | 60.71  | 61.22  | 61.74  | 62.05  | 62.26  | 62.53  | 62.69  | 62.79  | 63.06  | 63.30  |
| 241.88                              | 243.91 | 245.95 | 248.01 | 249.26 | 250.10 | 251.14 | 251.77 | 252.20 | 253.25 | 254.19 |
| 968.63                              | 976.71 | 984.87 | 993.10 | 998.08 | 1001.4 | 1005.6 | 1008.1 | 1009.8 | 1014.0 | 1017.7 |
| 6055.8                              | 6106.3 | 6157.3 | 6208.7 | 6239.8 | 6260.6 | 6286.8 | 6302.5 | 6313.0 | 6339.4 | 6362.7 |
| 605621                              | 610668 | 615764 | 620908 | 624017 | 626099 | 628712 | 630285 | 631337 | 633972 | 636301 |
| 9.39                                | 9.41   | 9.42   | 9.44   | 9.45   | 9.46   | 9.47   | 9.47   | 9.47   | 9.48   | 9.49   |
| 19.40                               | 19.41  | 19.43  | 19.45  | 19.46  | 19.46  | 19.47  | 19.48  | 19.48  | 19.49  | 19.49  |
| 39.40                               | 39.41  | 39.43  | 39.45  | 39.46  | 39.46  | 39.47  | 39.48  | 39.48  | 39.49  | 39.50  |
| 99.40                               | 99.42  | 99.43  | 99.45  | 99.46  | 99.47  | 99.47  | 99.48  | 99.48  | 99.49  | 99.50  |
| 999.40                              | 999.42 | 999.43 | 999.45 | 999.46 | 999.47 | 999.47 | 999.48 | 999.48 | 999.49 | 999.50 |
| 5.23                                | 5.22   | 5.20   | 5.18   | 5.17   | 5.17   | 5.16   | 5.15   | 5.15   | 5.14   | 5.13   |
| 8.79                                | 8.74   | 8.70   | 8.66   | 8.63   | 8.62   | 8.59   | 8.58   | 8.57   | 8.55   | 8.53   |
| 14.42                               | 14.34  | 14.25  | 14.17  | 14.12  | 14.08  | 14.04  | 14.01  | 13.99  | 13.95  | 13.91  |
| 27.23                               | 27.05  | 26.87  | 26.69  | 26.58  | 26.50  | 26.41  | 26.35  | 26.32  | 26.22  | 26.14  |
| 129.25                              | 128.32 | 127.37 | 126.42 | 125.84 | 125.45 | 124.96 | 124.66 | 124.47 | 123.97 | 123.53 |
| 3.92                                | 3.90   | 3.87   | 3.84   | 3.83   | 3.82   | 3.80   | 3.80   | 3.79   | 3.78   | 3.76   |
| 5.96                                | 5.91   | 5.86   | 5.80   | 5.77   | 5.75   | 5.72   | 5.70   | 5.69   | 5.66   | 5.63   |
| 8.84                                | 8.75   | 8.66   | 8.56   | 8.50   | 8.46   | 8.41   | 8.38   | 8.36   | 8.31   | 8.26   |
| 14.55                               | 14.37  | 14.20  | 14.02  | 13.91  | 13.84  | 13.75  | 13.69  | 13.65  | 13.56  | 13.47  |
| 48.05                               | 47.41  | 46.76  | 46.10  | 45.70  | 45.43  | 45.09  | 44.88  | 44.75  | 44.40  | 44.09  |
| 3.30                                | 3.27   | 3.24   | 3.21   | 3.19   | 3.17   | 3.16   | 3.15   | 3.14   | 3.12   | 3.11   |
| 4.74                                | 4.68   | 4.62   | 4.56   | 4.52   | 4.50   | 4.46   | 4.44   | 4.43   | 4.40   | 4.37   |
| 6.62                                | 6.52   | 6.43   | 6.33   | 6.27   | 6.23   | 6.18   | 6.14   | 6.12   | 6.07   | 6.02   |
| 10.05                               | 9.89   | 9.72   | 9.55   | 9.45   | 9.38   | 9.29   | 9.24   | 9.20   | 9.11   | 9.03   |
| 26.92                               | 26.42  | 25.91  | 25.39  | 25.08  | 24.87  | 24.60  | 24.44  | 24.33  | 24.06  | 23.82  |
| 2.94                                | 2.90   | 2.87   | 2.84   | 2.81   | 2.80   | 2.78   | 2.77   | 2.76   | 2.74   | 2.72   |
| 4.06                                | 4.00   | 3.94   | 3.87   | 3.83   | 3.81   | 3.77   | 3.75   | 3.74   | 3.70   | 3.67   |
| 5.46                                | 5.37   | 5.27   | 5.17   | 5.11   | 5.07   | 5.01   | 4.98   | 4.96   | 4.90   | 4.86   |
| 7.87                                | 7.72   | 7.56   | 7.40   | 7.30   | 7.23   | 7.14   | 7.09   | 7.06   | 6.97   | 6.89   |
| 18.41                               | 17.99  | 17.56  | 17.12  | 16.85  | 16.67  | 16.44  | 16.31  | 16.21  | 15.98  | 15.77  |
| 2.70                                | 2.67   | 2.63   | 2.59   | 2.57   | 2.56   | 2.54   | 2.52   | 2.51   | 2.49   | 2.47   |
| 3.64                                | 3.57   | 3.51   | 3.44   | 3.40   | 3.38   | 3.34   | 3.32   | 3.30   | 3.27   | 3.23   |
| 4.76                                | 4.67   | 4.57   | 4.47   | 4.40   | 4.36   | 4.31   | 4.28   | 4.25   | 4.20   | 4.15   |
| 6.62                                | 6.47   | 6.31   | 6.16   | 6.06   | 5.99   | 5.91   | 5.86   | 5.82   | 5.74   | 5.66   |
| 14.08                               | 13.71  | 13.32  | 12.93  | 12.69  | 12.53  | 12.33  | 12.20  | 12.12  | 11.91  | 11.72  |

| TABLE E                               |      |                                     |       |       |       |       |       |       |       |       |       |
|---------------------------------------|------|-------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| F critical values (continued)         |      |                                     |       |       |       |       |       |       |       |       |       |
|                                       |      | Degrees of freedom in the numerator |       |       |       |       |       |       |       |       |       |
| <i>p</i>                              |      | 1                                   | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |       |
| Degrees of freedom in the denominator | 8    | .100                                | 3.46  | 3.11  | 2.92  | 2.81  | 2.73  | 2.67  | 2.62  | 2.59  | 2.56  |
|                                       |      | .050                                | 5.32  | 4.46  | 4.07  | 3.84  | 3.69  | 3.58  | 3.50  | 3.44  | 3.39  |
|                                       |      | .025                                | 7.57  | 6.06  | 5.42  | 5.05  | 4.82  | 4.65  | 4.53  | 4.43  | 4.36  |
|                                       |      | .010                                | 11.26 | 8.65  | 7.59  | 7.01  | 6.63  | 6.37  | 6.18  | 6.03  | 5.91  |
|                                       |      | .001                                | 25.41 | 18.49 | 15.83 | 14.39 | 13.48 | 12.86 | 12.40 | 12.05 | 11.77 |
|                                       | 9    | .100                                | 3.36  | 3.01  | 2.81  | 2.69  | 2.61  | 2.55  | 2.51  | 2.47  | 2.44  |
|                                       |      | .050                                | 5.12  | 4.26  | 3.86  | 3.63  | 3.48  | 3.37  | 3.29  | 3.23  | 3.18  |
|                                       |      | .025                                | 7.21  | 5.71  | 5.08  | 4.72  | 4.48  | 4.32  | 4.20  | 4.10  | 4.03  |
|                                       |      | .010                                | 10.56 | 8.02  | 6.99  | 6.42  | 6.06  | 5.80  | 5.61  | 5.47  | 5.35  |
|                                       |      | .001                                | 22.86 | 16.39 | 13.90 | 12.56 | 11.71 | 11.13 | 10.70 | 10.37 | 10.11 |
|                                       | 10   | .100                                | 3.29  | 2.92  | 2.73  | 2.61  | 2.52  | 2.46  | 2.41  | 2.38  | 2.35  |
|                                       |      | .050                                | 4.96  | 4.10  | 3.71  | 3.48  | 3.33  | 3.22  | 3.14  | 3.07  | 3.02  |
|                                       |      | .025                                | 6.94  | 5.46  | 4.83  | 4.47  | 4.24  | 4.07  | 3.95  | 3.85  | 3.78  |
|                                       |      | .010                                | 10.04 | 7.56  | 6.55  | 5.99  | 5.64  | 5.39  | 5.20  | 5.06  | 4.94  |
|                                       |      | .001                                | 21.04 | 14.91 | 12.55 | 11.28 | 10.48 | 9.93  | 9.52  | 9.20  | 8.96  |
|                                       | 11   | .100                                | 3.23  | 2.86  | 2.66  | 2.54  | 2.45  | 2.39  | 2.34  | 2.30  | 2.27  |
|                                       |      | .050                                | 4.84  | 3.98  | 3.59  | 3.36  | 3.20  | 3.09  | 3.01  | 2.95  | 2.90  |
|                                       |      | .025                                | 6.72  | 5.26  | 4.63  | 4.28  | 4.04  | 3.88  | 3.76  | 3.66  | 3.59  |
|                                       |      | .010                                | 9.65  | 7.21  | 6.22  | 5.67  | 5.32  | 5.07  | 4.89  | 4.74  | 4.63  |
|                                       |      | .001                                | 19.69 | 13.81 | 11.56 | 10.35 | 9.58  | 9.05  | 8.66  | 8.35  | 8.12  |
| 12                                    | .100 | 3.18                                | 2.81  | 2.61  | 2.48  | 2.39  | 2.33  | 2.28  | 2.24  | 2.21  |       |
|                                       | .050 | 4.75                                | 3.89  | 3.49  | 3.26  | 3.11  | 3.00  | 2.91  | 2.85  | 2.80  |       |
|                                       | .025 | 6.55                                | 5.10  | 4.47  | 4.12  | 3.89  | 3.73  | 3.61  | 3.51  | 3.44  |       |
|                                       | .010 | 9.33                                | 6.93  | 5.95  | 5.41  | 5.06  | 4.82  | 4.64  | 4.50  | 4.39  |       |
|                                       | .001 | 18.64                               | 12.97 | 10.80 | 9.63  | 8.89  | 8.38  | 8.00  | 7.71  | 7.48  |       |
| 13                                    | .100 | 3.14                                | 2.76  | 2.56  | 2.43  | 2.35  | 2.28  | 2.23  | 2.20  | 2.16  |       |
|                                       | .050 | 4.67                                | 3.81  | 3.41  | 3.18  | 3.03  | 2.92  | 2.83  | 2.77  | 2.71  |       |
|                                       | .025 | 6.41                                | 4.97  | 4.35  | 4.00  | 3.77  | 3.60  | 3.48  | 3.39  | 3.31  |       |
|                                       | .010 | 9.07                                | 6.70  | 5.74  | 5.21  | 4.86  | 4.62  | 4.44  | 4.30  | 4.19  |       |
|                                       | .001 | 17.82                               | 12.31 | 10.21 | 9.07  | 8.35  | 7.86  | 7.49  | 7.21  | 6.98  |       |
| 14                                    | .100 | 3.10                                | 2.73  | 2.52  | 2.39  | 2.31  | 2.24  | 2.19  | 2.15  | 2.12  |       |
|                                       | .050 | 4.60                                | 3.74  | 3.34  | 3.11  | 2.96  | 2.85  | 2.76  | 2.70  | 2.65  |       |
|                                       | .025 | 6.30                                | 4.86  | 4.24  | 3.89  | 3.66  | 3.50  | 3.38  | 3.29  | 3.21  |       |
|                                       | .010 | 8.86                                | 6.51  | 5.56  | 5.04  | 4.69  | 4.46  | 4.28  | 4.14  | 4.03  |       |
|                                       | .001 | 17.14                               | 11.78 | 9.73  | 8.62  | 7.92  | 7.44  | 7.08  | 6.80  | 6.58  |       |
| 15                                    | .100 | 3.07                                | 2.70  | 2.49  | 2.36  | 2.27  | 2.21  | 2.16  | 2.12  | 2.09  |       |
|                                       | .050 | 4.54                                | 3.68  | 3.29  | 3.06  | 2.90  | 2.79  | 2.71  | 2.64  | 2.59  |       |
|                                       | .025 | 6.20                                | 4.77  | 4.15  | 3.80  | 3.58  | 3.41  | 3.29  | 3.20  | 3.12  |       |
|                                       | .010 | 8.68                                | 6.36  | 5.42  | 4.89  | 4.56  | 4.32  | 4.14  | 4.00  | 3.89  |       |
|                                       | .001 | 16.59                               | 11.34 | 9.34  | 8.25  | 7.57  | 7.09  | 6.74  | 6.47  | 6.26  |       |
| 16                                    | .100 | 3.05                                | 2.67  | 2.46  | 2.33  | 2.24  | 2.18  | 2.13  | 2.09  | 2.06  |       |
|                                       | .050 | 4.49                                | 3.63  | 3.24  | 3.01  | 2.85  | 2.74  | 2.66  | 2.59  | 2.54  |       |
|                                       | .025 | 6.12                                | 4.69  | 4.08  | 3.73  | 3.50  | 3.34  | 3.22  | 3.12  | 3.05  |       |
|                                       | .010 | 8.53                                | 6.23  | 5.29  | 4.77  | 4.44  | 4.20  | 4.03  | 3.89  | 3.78  |       |
|                                       | .001 | 16.12                               | 10.97 | 9.01  | 7.94  | 7.27  | 6.80  | 6.46  | 6.19  | 5.98  |       |
| 17                                    | .100 | 3.03                                | 2.64  | 2.44  | 2.31  | 2.22  | 2.15  | 2.10  | 2.06  | 2.03  |       |
|                                       | .050 | 4.45                                | 3.59  | 3.20  | 2.96  | 2.81  | 2.70  | 2.61  | 2.55  | 2.49  |       |
|                                       | .025 | 6.04                                | 4.62  | 4.01  | 3.66  | 3.44  | 3.28  | 3.16  | 3.06  | 2.98  |       |
|                                       | .010 | 8.40                                | 6.11  | 5.19  | 4.67  | 4.34  | 4.10  | 3.93  | 3.79  | 3.68  |       |
|                                       | .001 | 15.72                               | 10.66 | 8.73  | 7.68  | 7.02  | 6.56  | 6.22  | 5.96  | 5.75  |       |

| TABLE E                               |    |                                     |       |       |       |      |      |      |      |      |      |      |
|---------------------------------------|----|-------------------------------------|-------|-------|-------|------|------|------|------|------|------|------|
| F critical values (continued)         |    |                                     |       |       |       |      |      |      |      |      |      |      |
|                                       |    | Degrees of freedom in the numerator |       |       |       |      |      |      |      |      |      |      |
| <i>p</i>                              |    | 1                                   | 2     | 3     | 4     | 5    | 6    | 7    | 8    | 9    |      |      |
| Degrees of freedom in the denominator | 18 | .100                                | 3.01  | 2.62  | 2.42  | 2.29 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 |      |
|                                       |    | .050                                | 4.41  | 3.55  | 3.16  | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 |      |
|                                       |    | .025                                | 5.98  | 4.56  | 3.95  | 3.61 | 3.38 | 3.22 | 3.10 | 3.01 | 2.93 |      |
|                                       |    | .010                                | 8.29  | 6.01  | 5.09  | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 |      |
|                                       |    | .001                                | 15.38 | 10.39 | 8.49  | 7.46 | 6.81 | 6.35 | 6.02 | 5.76 | 5.56 |      |
|                                       |    | 19                                  | .100  | 2.99  | 2.61  | 2.40 | 2.27 | 2.18 | 2.11 | 2.06 | 2.02 | 1.98 |
|                                       |    |                                     | .050  | 4.38  | 3.52  | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 |
|                                       |    |                                     | .025  | 5.92  | 4.51  | 3.90 | 3.56 | 3.33 | 3.17 | 3.05 | 2.96 | 2.88 |
|                                       |    |                                     | .010  | 8.18  | 5.93  | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 |
|                                       |    |                                     | .001  | 15.08 | 10.16 | 8.28 | 7.27 | 6.62 | 6.18 | 5.85 | 5.59 | 5.39 |
|                                       |    | 20                                  | .100  | 2.97  | 2.59  | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 |
|                                       |    |                                     | .050  | 4.35  | 3.49  | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 |
|                                       |    |                                     | .025  | 5.87  | 4.46  | 3.86 | 3.51 | 3.29 | 3.13 | 3.01 | 2.91 | 2.84 |
|                                       |    |                                     | .010  | 8.10  | 5.85  | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 |
|                                       |    |                                     | .001  | 14.82 | 9.95  | 8.10 | 7.10 | 6.46 | 6.02 | 5.69 | 5.44 | 5.24 |
|                                       |    | 21                                  | .100  | 2.96  | 2.57  | 2.36 | 2.23 | 2.14 | 2.08 | 2.02 | 1.98 | 1.95 |
|                                       |    |                                     | .050  | 4.32  | 3.47  | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 |
|                                       |    |                                     | .025  | 5.83  | 4.42  | 3.82 | 3.48 | 3.25 | 3.09 | 2.97 | 2.87 | 2.80 |
|                                       |    |                                     | .010  | 8.02  | 5.78  | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 |
|                                       |    |                                     | .001  | 14.59 | 9.77  | 7.94 | 6.95 | 6.32 | 5.88 | 5.56 | 5.31 | 5.11 |
|                                       | 22 | .100                                | 2.95  | 2.56  | 2.35  | 2.22 | 2.13 | 2.06 | 2.01 | 1.97 | 1.93 |      |
|                                       |    | .050                                | 4.30  | 3.44  | 3.05  | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 |      |
|                                       |    | .025                                | 5.79  | 4.38  | 3.78  | 3.44 | 3.22 | 3.05 | 2.93 | 2.84 | 2.76 |      |
|                                       |    | .010                                | 7.95  | 5.72  | 4.82  | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 |      |
|                                       |    | .001                                | 14.38 | 9.61  | 7.80  | 6.81 | 6.19 | 5.76 | 5.44 | 5.19 | 4.99 |      |
|                                       | 23 | .100                                | 2.94  | 2.55  | 2.34  | 2.21 | 2.11 | 2.05 | 1.99 | 1.95 | 1.92 |      |
|                                       |    | .050                                | 4.28  | 3.42  | 3.03  | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 |      |
|                                       |    | .025                                | 5.75  | 4.35  | 3.75  | 3.41 | 3.18 | 3.02 | 2.90 | 2.81 | 2.73 |      |
|                                       |    | .010                                | 7.88  | 5.66  | 4.76  | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 |      |
|                                       |    | .001                                | 14.20 | 9.47  | 7.67  | 6.70 | 6.08 | 5.65 | 5.33 | 5.09 | 4.89 |      |
|                                       | 24 | .100                                | 2.93  | 2.54  | 2.33  | 2.19 | 2.10 | 2.04 | 1.98 | 1.94 | 1.91 |      |
|                                       |    | .050                                | 4.26  | 3.40  | 3.01  | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 |      |
|                                       |    | .025                                | 5.72  | 4.32  | 3.72  | 3.38 | 3.15 | 2.99 | 2.87 | 2.78 | 2.70 |      |
|                                       |    | .010                                | 7.82  | 5.61  | 4.72  | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 |      |
|                                       |    | .001                                | 14.03 | 9.34  | 7.55  | 6.59 | 5.98 | 5.55 | 5.23 | 4.99 | 4.80 |      |
|                                       | 25 | .100                                | 2.92  | 2.53  | 2.32  | 2.18 | 2.09 | 2.02 | 1.97 | 1.93 | 1.89 |      |
|                                       |    | .050                                | 4.24  | 3.39  | 2.99  | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 |      |
|                                       |    | .025                                | 5.69  | 4.29  | 3.69  | 3.35 | 3.13 | 2.97 | 2.85 | 2.75 | 2.68 |      |
|                                       |    | .010                                | 7.77  | 5.57  | 4.68  | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 |      |
|                                       |    | .001                                | 13.88 | 9.22  | 7.45  | 6.49 | 5.89 | 5.46 | 5.15 | 4.91 | 4.71 |      |
|                                       | 26 | .100                                | 2.91  | 2.52  | 2.31  | 2.17 | 2.08 | 2.01 | 1.96 | 1.92 | 1.88 |      |
|                                       |    | .050                                | 4.23  | 3.37  | 2.98  | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 |      |
|                                       |    | .025                                | 5.66  | 4.27  | 3.67  | 3.33 | 3.10 | 2.94 | 2.82 | 2.73 | 2.65 |      |
|                                       |    | .010                                | 7.72  | 5.53  | 4.64  | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 |      |
|                                       |    | .001                                | 13.74 | 9.12  | 7.36  | 6.41 | 5.80 | 5.38 | 5.07 | 4.83 | 4.64 |      |
|                                       | 27 | .100                                | 2.90  | 2.51  | 2.30  | 2.17 | 2.07 | 2.00 | 1.95 | 1.91 | 1.87 |      |
|                                       |    | .050                                | 4.21  | 3.35  | 2.96  | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 |      |
|                                       |    | .025                                | 5.63  | 4.24  | 3.65  | 3.31 | 3.08 | 2.92 | 2.80 | 2.71 | 2.63 |      |
|                                       |    | .010                                | 7.68  | 5.49  | 4.60  | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 |      |
|                                       |    | .001                                | 13.61 | 9.02  | 7.27  | 6.33 | 5.73 | 5.31 | 5.00 | 4.76 | 4.57 |      |



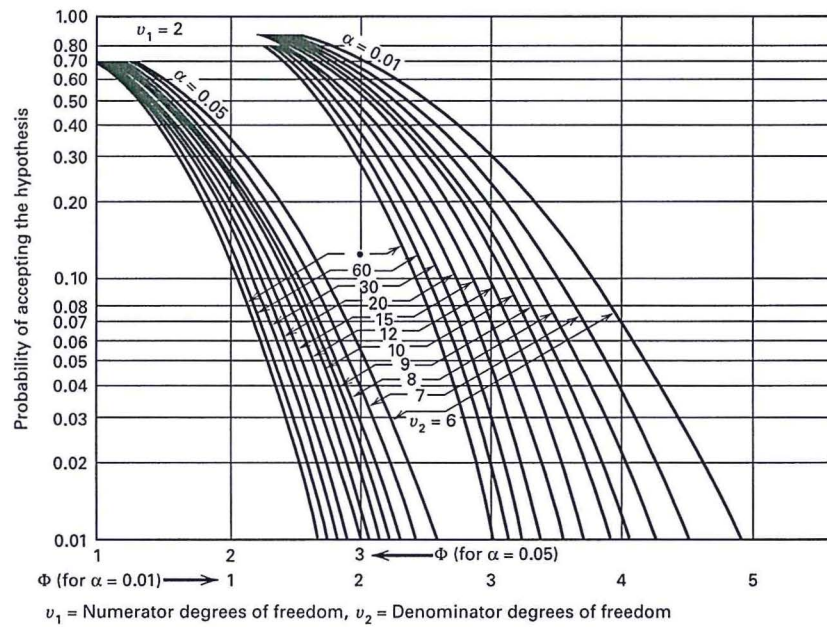
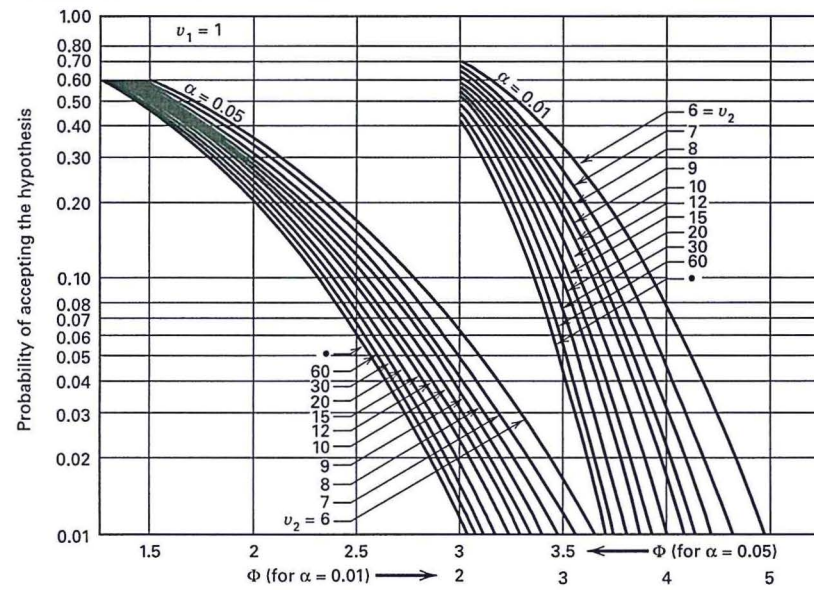
**TABLE E**

*F* critical values (continued)

| Degrees of freedom in the numerator |       |       |       |       |       |      |      |      |      |      |
|-------------------------------------|-------|-------|-------|-------|-------|------|------|------|------|------|
| 10                                  | 12    | 15    | 20    | 25    | 30    | 40   | 50   | 60   | 120  | 1000 |
| 2.54                                | 2.50  | 2.46  | 2.42  | 2.40  | 2.38  | 2.36 | 2.35 | 2.34 | 2.32 | 2.30 |
| 3.35                                | 3.28  | 3.22  | 3.15  | 3.11  | 3.08  | 3.04 | 3.02 | 3.01 | 2.97 | 2.93 |
| 4.30                                | 4.20  | 4.10  | 4.00  | 3.94  | 3.89  | 3.84 | 3.81 | 3.78 | 3.73 | 3.68 |
| 5.81                                | 5.67  | 5.52  | 5.36  | 5.26  | 5.20  | 5.12 | 5.07 | 5.03 | 4.95 | 4.87 |
| 11.54                               | 11.19 | 10.84 | 10.48 | 10.26 | 10.11 | 9.92 | 9.80 | 9.73 | 9.53 | 9.36 |
| 2.42                                | 2.38  | 2.34  | 2.30  | 2.27  | 2.25  | 2.23 | 2.22 | 2.21 | 2.18 | 2.16 |
| 3.14                                | 3.07  | 3.01  | 2.94  | 2.89  | 2.86  | 2.83 | 2.80 | 2.79 | 2.75 | 2.71 |
| 3.96                                | 3.87  | 3.77  | 3.67  | 3.60  | 3.56  | 3.51 | 3.47 | 3.45 | 3.39 | 3.34 |
| 5.26                                | 5.11  | 4.96  | 4.81  | 4.71  | 4.65  | 4.57 | 4.52 | 4.48 | 4.40 | 4.32 |
| 9.89                                | 9.57  | 9.24  | 8.90  | 8.69  | 8.55  | 8.37 | 8.26 | 8.19 | 8.00 | 7.84 |
| 2.32                                | 2.28  | 2.24  | 2.20  | 2.17  | 2.16  | 2.13 | 2.12 | 2.11 | 2.08 | 2.06 |
| 2.98                                | 2.91  | 2.85  | 2.77  | 2.73  | 2.70  | 2.66 | 2.64 | 2.62 | 2.58 | 2.54 |
| 3.72                                | 3.62  | 3.52  | 3.42  | 3.35  | 3.31  | 3.26 | 3.22 | 3.20 | 3.14 | 3.09 |
| 4.85                                | 4.71  | 4.56  | 4.41  | 4.31  | 4.25  | 4.17 | 4.12 | 4.08 | 4.00 | 3.92 |
| 8.75                                | 8.45  | 8.13  | 7.80  | 7.60  | 7.47  | 7.30 | 7.19 | 7.12 | 6.94 | 6.78 |
| 2.25                                | 2.21  | 2.17  | 2.12  | 2.10  | 2.08  | 2.05 | 2.04 | 2.03 | 2.00 | 1.98 |
| 2.85                                | 2.79  | 2.72  | 2.65  | 2.60  | 2.57  | 2.53 | 2.51 | 2.49 | 2.45 | 2.41 |
| 3.53                                | 3.43  | 3.33  | 3.23  | 3.16  | 3.12  | 3.06 | 3.03 | 3.00 | 2.94 | 2.89 |
| 4.54                                | 4.40  | 4.25  | 4.10  | 4.01  | 3.94  | 3.86 | 3.81 | 3.78 | 3.69 | 3.61 |
| 7.92                                | 7.63  | 7.32  | 7.01  | 6.81  | 6.68  | 6.52 | 6.42 | 6.35 | 6.18 | 6.02 |
| 2.19                                | 2.15  | 2.10  | 2.06  | 2.03  | 2.01  | 1.99 | 1.97 | 1.96 | 1.93 | 1.91 |
| 2.75                                | 2.69  | 2.62  | 2.54  | 2.50  | 2.47  | 2.43 | 2.40 | 2.38 | 2.34 | 2.30 |
| 3.37                                | 3.28  | 3.18  | 3.07  | 3.01  | 2.96  | 2.91 | 2.87 | 2.85 | 2.79 | 2.73 |
| 4.30                                | 4.16  | 4.01  | 3.86  | 3.76  | 3.70  | 3.62 | 3.57 | 3.54 | 3.45 | 3.37 |
| 7.29                                | 7.00  | 6.71  | 6.40  | 6.22  | 6.09  | 5.93 | 5.83 | 5.76 | 5.59 | 5.44 |
| 2.14                                | 2.10  | 2.05  | 2.01  | 1.98  | 1.96  | 1.93 | 1.92 | 1.90 | 1.88 | 1.85 |
| 2.67                                | 2.60  | 2.53  | 2.46  | 2.41  | 2.38  | 2.34 | 2.31 | 2.30 | 2.25 | 2.21 |
| 3.25                                | 3.15  | 3.05  | 2.95  | 2.88  | 2.84  | 2.78 | 2.74 | 2.72 | 2.66 | 2.60 |
| 4.10                                | 3.96  | 3.82  | 3.66  | 3.57  | 3.51  | 3.43 | 3.38 | 3.34 | 3.25 | 3.18 |
| 6.80                                | 6.52  | 6.23  | 5.93  | 5.75  | 5.63  | 5.47 | 5.37 | 5.30 | 5.14 | 4.99 |
| 2.10                                | 2.05  | 2.01  | 1.96  | 1.93  | 1.91  | 1.89 | 1.87 | 1.86 | 1.83 | 1.80 |
| 2.60                                | 2.53  | 2.46  | 2.39  | 2.34  | 2.31  | 2.27 | 2.24 | 2.22 | 2.18 | 2.14 |
| 3.15                                | 3.05  | 2.95  | 2.84  | 2.78  | 2.73  | 2.67 | 2.64 | 2.61 | 2.55 | 2.50 |
| 3.94                                | 3.80  | 3.66  | 3.51  | 3.41  | 3.35  | 3.27 | 3.22 | 3.18 | 3.09 | 3.02 |
| 6.40                                | 6.13  | 5.85  | 5.56  | 5.38  | 5.25  | 5.10 | 5.00 | 4.94 | 4.77 | 4.62 |
| 2.06                                | 2.02  | 1.97  | 1.92  | 1.89  | 1.87  | 1.85 | 1.83 | 1.82 | 1.79 | 1.76 |
| 2.54                                | 2.48  | 2.40  | 2.33  | 2.28  | 2.25  | 2.20 | 2.18 | 2.16 | 2.11 | 2.07 |
| 3.06                                | 2.96  | 2.86  | 2.76  | 2.69  | 2.64  | 2.59 | 2.55 | 2.52 | 2.46 | 2.40 |
| 3.80                                | 3.67  | 3.52  | 3.37  | 3.28  | 3.21  | 3.13 | 3.08 | 3.05 | 2.96 | 2.88 |
| 6.08                                | 5.81  | 5.54  | 5.25  | 5.07  | 4.95  | 4.80 | 4.70 | 4.64 | 4.47 | 4.33 |
| 2.03                                | 1.99  | 1.94  | 1.89  | 1.86  | 1.84  | 1.81 | 1.79 | 1.78 | 1.75 | 1.72 |
| 2.49                                | 2.42  | 2.35  | 2.28  | 2.23  | 2.19  | 2.15 | 2.12 | 2.11 | 2.06 | 2.02 |
| 2.99                                | 2.89  | 2.79  | 2.68  | 2.61  | 2.57  | 2.51 | 2.47 | 2.45 | 2.38 | 2.32 |
| 3.69                                | 3.55  | 3.41  | 3.26  | 3.16  | 3.10  | 3.02 | 2.97 | 2.93 | 2.84 | 2.76 |
| 5.81                                | 5.55  | 5.27  | 4.99  | 4.82  | 4.70  | 4.54 | 4.45 | 4.39 | 4.23 | 4.08 |
| 2.00                                | 1.96  | 1.91  | 1.86  | 1.83  | 1.81  | 1.78 | 1.76 | 1.75 | 1.72 | 1.69 |
| 2.45                                | 2.38  | 2.31  | 2.23  | 2.18  | 2.15  | 2.10 | 2.08 | 2.06 | 2.01 | 1.97 |
| 2.92                                | 2.82  | 2.72  | 2.62  | 2.55  | 2.50  | 2.44 | 2.41 | 2.38 | 2.32 | 2.26 |
| 3.59                                | 3.46  | 3.31  | 3.16  | 3.07  | 3.00  | 2.92 | 2.87 | 2.83 | 2.75 | 2.66 |
| 5.58                                | 5.32  | 5.05  | 4.78  | 4.60  | 4.48  | 4.33 | 4.24 | 4.18 | 4.02 | 3.87 |

| TABLE E                             |      |      |      |      |      |      |      |      |      |      |
|-------------------------------------|------|------|------|------|------|------|------|------|------|------|
| F critical values (continued)       |      |      |      |      |      |      |      |      |      |      |
| Degrees of freedom in the numerator |      |      |      |      |      |      |      |      |      |      |
| 10                                  | 12   | 15   | 20   | 25   | 30   | 40   | 50   | 60   | 120  | 1000 |
| 1.98                                | 1.93 | 1.89 | 1.84 | 1.80 | 1.78 | 1.75 | 1.74 | 1.72 | 1.69 | 1.66 |
| 2.41                                | 2.34 | 2.27 | 2.19 | 2.14 | 2.11 | 2.06 | 2.04 | 2.02 | 1.97 | 1.92 |
| 2.87                                | 2.77 | 2.67 | 2.56 | 2.49 | 2.44 | 2.38 | 2.35 | 2.32 | 2.26 | 2.20 |
| 3.51                                | 3.37 | 3.23 | 3.08 | 2.98 | 2.92 | 2.84 | 2.78 | 2.75 | 2.66 | 2.58 |
| 5.39                                | 5.13 | 4.87 | 4.59 | 4.42 | 4.30 | 4.15 | 4.06 | 4.00 | 3.84 | 3.69 |
| 1.96                                | 1.91 | 1.86 | 1.81 | 1.78 | 1.76 | 1.73 | 1.71 | 1.70 | 1.67 | 1.64 |
| 2.38                                | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 2.00 | 1.98 | 1.93 | 1.88 |
| 2.82                                | 2.72 | 2.62 | 2.51 | 2.44 | 2.39 | 2.33 | 2.30 | 2.27 | 2.20 | 2.14 |
| 3.43                                | 3.30 | 3.15 | 3.00 | 2.91 | 2.84 | 2.76 | 2.71 | 2.67 | 2.58 | 2.50 |
| 5.22                                | 4.97 | 4.70 | 4.43 | 4.26 | 4.14 | 3.99 | 3.90 | 3.84 | 3.68 | 3.53 |
| 1.94                                | 1.89 | 1.84 | 1.79 | 1.76 | 1.74 | 1.71 | 1.69 | 1.68 | 1.64 | 1.61 |
| 2.35                                | 2.28 | 2.20 | 2.12 | 2.07 | 2.04 | 1.99 | 1.97 | 1.95 | 1.90 | 1.85 |
| 2.77                                | 2.68 | 2.57 | 2.46 | 2.40 | 2.35 | 2.29 | 2.25 | 2.22 | 2.16 | 2.09 |
| 3.37                                | 3.23 | 3.09 | 2.94 | 2.84 | 2.78 | 2.69 | 2.64 | 2.61 | 2.52 | 2.43 |
| 5.08                                | 4.82 | 4.56 | 4.29 | 4.12 | 4.00 | 3.86 | 3.77 | 3.70 | 3.54 | 3.40 |
| 1.92                                | 1.87 | 1.83 | 1.78 | 1.74 | 1.72 | 1.69 | 1.67 | 1.66 | 1.62 | 1.59 |
| 2.32                                | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 | 1.94 | 1.92 | 1.87 | 1.82 |
| 2.73                                | 2.64 | 2.53 | 2.42 | 2.36 | 2.31 | 2.25 | 2.21 | 2.18 | 2.11 | 2.05 |
| 3.31                                | 3.17 | 3.03 | 2.88 | 2.79 | 2.72 | 2.64 | 2.58 | 2.55 | 2.46 | 2.37 |
| 4.95                                | 4.70 | 4.44 | 4.17 | 4.00 | 3.88 | 3.74 | 3.64 | 3.58 | 3.42 | 3.28 |
| 1.90                                | 1.86 | 1.81 | 1.76 | 1.73 | 1.70 | 1.67 | 1.65 | 1.64 | 1.60 | 1.57 |
| 2.30                                | 2.23 | 2.15 | 2.07 | 2.02 | 1.98 | 1.94 | 1.91 | 1.89 | 1.84 | 1.79 |
| 2.70                                | 2.60 | 2.50 | 2.39 | 2.32 | 2.27 | 2.21 | 2.17 | 2.14 | 2.08 | 2.01 |
| 3.26                                | 3.12 | 2.98 | 2.83 | 2.73 | 2.67 | 2.58 | 2.53 | 2.50 | 2.40 | 2.32 |
| 4.83                                | 4.58 | 4.33 | 4.06 | 3.89 | 3.78 | 3.63 | 3.54 | 3.48 | 3.32 | 3.17 |
| 1.89                                | 1.84 | 1.80 | 1.74 | 1.71 | 1.69 | 1.66 | 1.64 | 1.62 | 1.59 | 1.55 |
| 2.27                                | 2.20 | 2.13 | 2.05 | 2.00 | 1.96 | 1.91 | 1.88 | 1.86 | 1.81 | 1.76 |
| 2.67                                | 2.57 | 2.47 | 2.36 | 2.29 | 2.24 | 2.18 | 2.14 | 2.11 | 2.04 | 1.98 |
| 3.21                                | 3.07 | 2.93 | 2.78 | 2.69 | 2.62 | 2.54 | 2.48 | 2.45 | 2.35 | 2.27 |
| 4.73                                | 4.48 | 4.23 | 3.96 | 3.79 | 3.68 | 3.53 | 3.44 | 3.38 | 3.22 | 3.08 |
| 1.88                                | 1.83 | 1.78 | 1.73 | 1.70 | 1.67 | 1.64 | 1.62 | 1.61 | 1.57 | 1.54 |
| 2.25                                | 2.18 | 2.11 | 2.03 | 1.97 | 1.94 | 1.89 | 1.86 | 1.84 | 1.79 | 1.74 |
| 2.64                                | 2.54 | 2.44 | 2.33 | 2.26 | 2.21 | 2.15 | 2.11 | 2.08 | 2.01 | 1.94 |
| 3.17                                | 3.03 | 2.89 | 2.74 | 2.64 | 2.58 | 2.49 | 2.44 | 2.40 | 2.31 | 2.22 |
| 4.64                                | 4.39 | 4.14 | 3.87 | 3.71 | 3.59 | 3.45 | 3.36 | 3.29 | 3.14 | 2.99 |
| 1.87                                | 1.82 | 1.77 | 1.72 | 1.68 | 1.66 | 1.63 | 1.61 | 1.59 | 1.56 | 1.52 |
| 2.24                                | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 | 1.84 | 1.82 | 1.77 | 1.72 |
| 2.61                                | 2.51 | 2.41 | 2.30 | 2.23 | 2.18 | 2.12 | 2.08 | 2.05 | 1.98 | 1.91 |
| 3.13                                | 2.99 | 2.85 | 2.70 | 2.60 | 2.54 | 2.45 | 2.40 | 2.36 | 2.27 | 2.18 |
| 4.56                                | 4.31 | 4.06 | 3.79 | 3.63 | 3.52 | 3.37 | 3.28 | 3.22 | 3.06 | 2.91 |
| 1.86                                | 1.81 | 1.76 | 1.71 | 1.67 | 1.65 | 1.61 | 1.59 | 1.58 | 1.54 | 1.51 |
| 2.22                                | 2.15 | 2.07 | 1.99 | 1.94 | 1.90 | 1.85 | 1.82 | 1.80 | 1.75 | 1.70 |
| 2.59                                | 2.49 | 2.39 | 2.28 | 2.21 | 2.16 | 2.09 | 2.05 | 2.03 | 1.95 | 1.89 |
| 3.09                                | 2.96 | 2.81 | 2.66 | 2.57 | 2.50 | 2.42 | 2.36 | 2.33 | 2.23 | 2.14 |
| 4.48                                | 4.24 | 3.99 | 3.72 | 3.56 | 3.44 | 3.30 | 3.21 | 3.15 | 2.99 | 2.84 |
| 1.85                                | 1.80 | 1.75 | 1.70 | 1.66 | 1.64 | 1.60 | 1.58 | 1.57 | 1.53 | 1.50 |
| 2.20                                | 2.13 | 2.06 | 1.97 | 1.92 | 1.88 | 1.84 | 1.81 | 1.79 | 1.73 | 1.68 |
| 2.57                                | 2.47 | 2.36 | 2.25 | 2.18 | 2.13 | 2.07 | 2.03 | 2.00 | 1.93 | 1.86 |
| 3.06                                | 2.93 | 2.78 | 2.63 | 2.54 | 2.47 | 2.38 | 2.33 | 2.29 | 2.20 | 2.11 |
| 4.41                                | 4.17 | 3.92 | 3.66 | 3.49 | 3.38 | 3.23 | 3.14 | 3.08 | 2.92 | 2.78 |

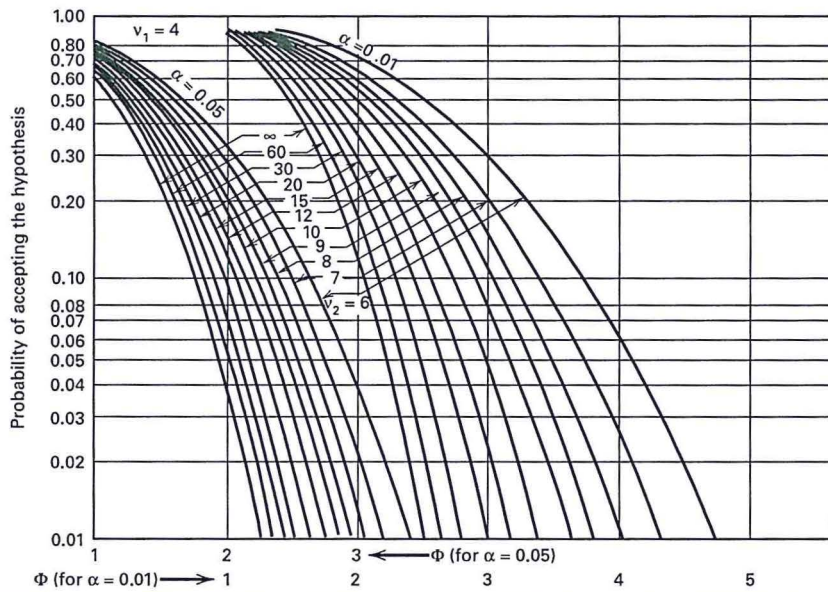
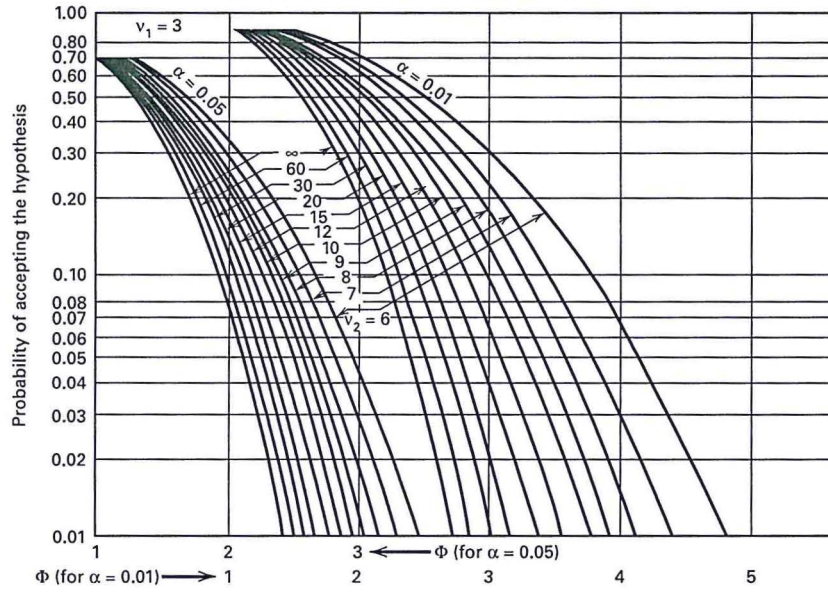
V Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance<sup>a</sup>



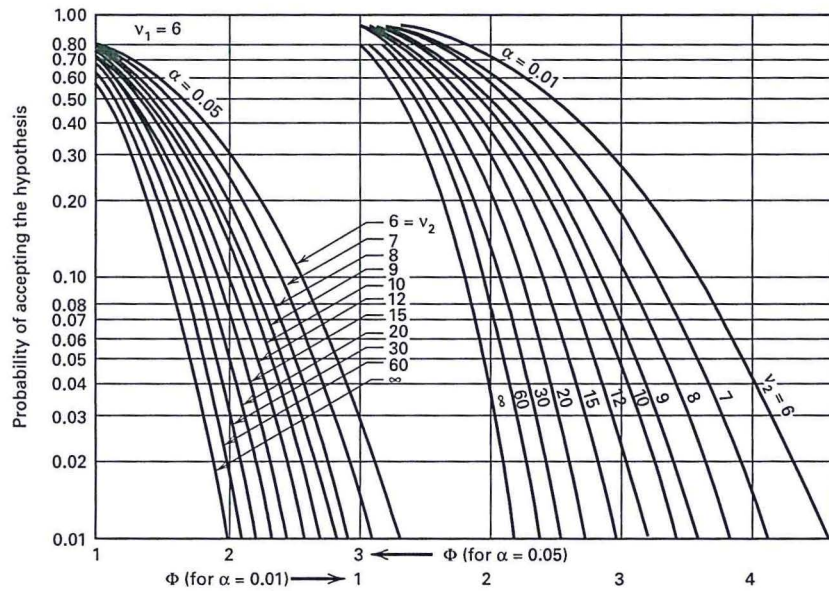
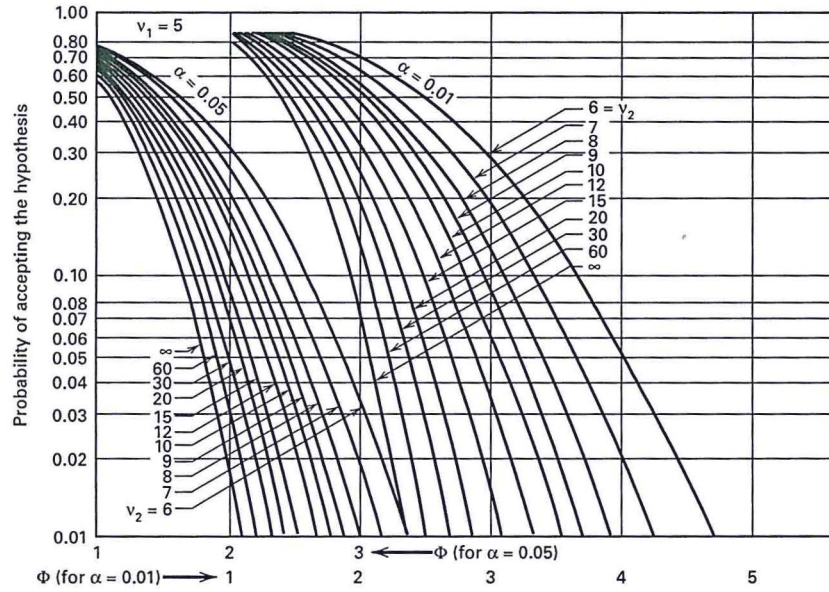
$v_1$  = Numerator degrees of freedom,  $v_2$  = Denominator degrees of freedom

<sup>a</sup>Adapted with permission from *Biometrika Tables for Statisticians*, Vol. 2, by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1972.

V Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance (Continued)



V Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance (Continued)



V Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance (Continued)

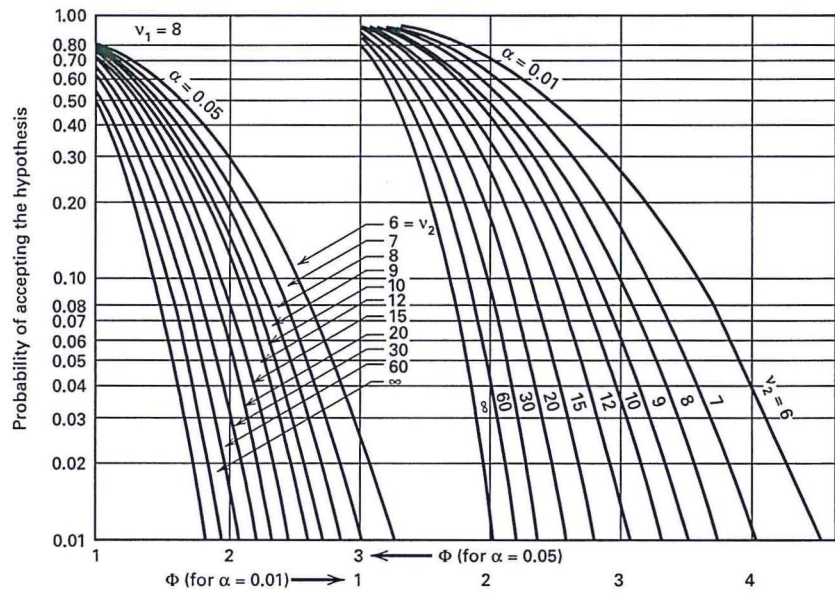
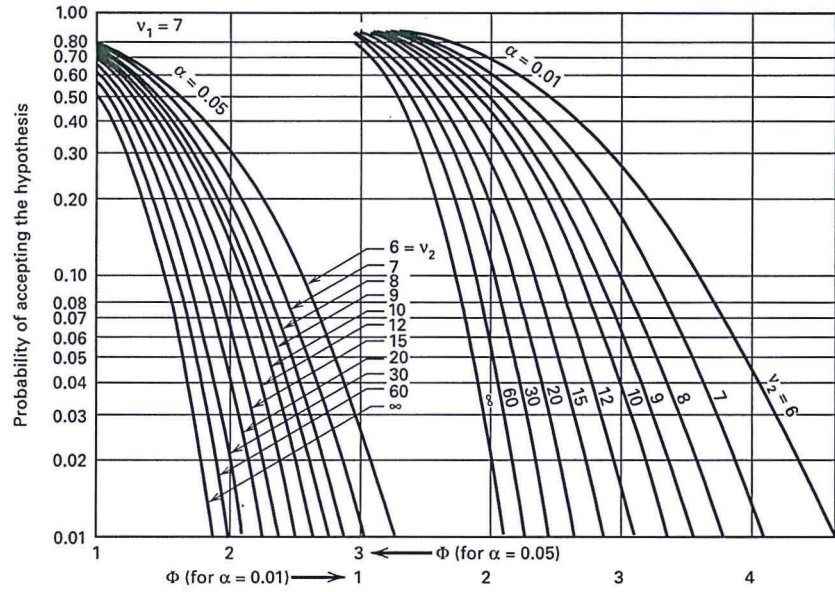
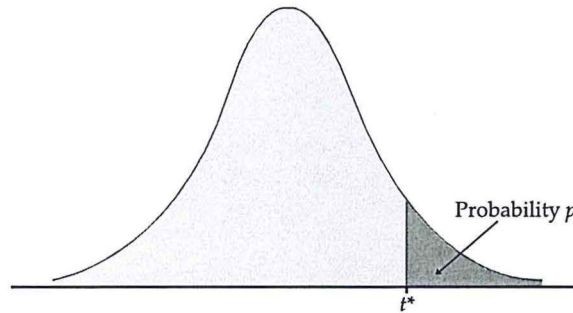


Table entry for  $p$  and  $C$  is the critical value  $t^*$  with probability  $p$  lying to its right and probability  $C$  lying between  $-t^*$  and  $t^*$ .



| TABLE D                        |                            |       |       |       |       |       |       |       |       |       |       |       |
|--------------------------------|----------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| t distribution critical values |                            |       |       |       |       |       |       |       |       |       |       |       |
| df                             | Upper-tail probability $p$ |       |       |       |       |       |       |       |       |       |       |       |
|                                | .25                        | .20   | .15   | .10   | .05   | .025  | .02   | .01   | .005  | .0025 | .001  | .0005 |
| 1                              | 1.000                      | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 15.89 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 2                              | 0.816                      | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 3                              | 0.765                      | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 4                              | 0.741                      | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5                              | 0.727                      | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 |
| 6                              | 0.718                      | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7                              | 0.711                      | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8                              | 0.706                      | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9                              | 0.703                      | 0.883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 10                             | 0.700                      | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11                             | 0.697                      | 0.876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12                             | 0.695                      | 0.873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13                             | 0.694                      | 0.870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14                             | 0.692                      | 0.868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15                             | 0.691                      | 0.866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16                             | 0.690                      | 0.865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17                             | 0.689                      | 0.863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18                             | 0.688                      | 0.862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.214 | 2.552 | 2.878 | 3.197 | 3.611 | 3.922 |
| 19                             | 0.688                      | 0.861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.205 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20                             | 0.687                      | 0.860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.197 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21                             | 0.686                      | 0.859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.189 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22                             | 0.686                      | 0.858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.183 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 23                             | 0.685                      | 0.858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.177 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24                             | 0.685                      | 0.857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.172 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25                             | 0.684                      | 0.856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.167 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26                             | 0.684                      | 0.856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.162 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27                             | 0.684                      | 0.855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.158 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 |
| 28                             | 0.683                      | 0.855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.154 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29                             | 0.683                      | 0.854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.150 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 |
| 30                             | 0.683                      | 0.854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.147 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 40                             | 0.681                      | 0.851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.123 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 50                             | 0.679                      | 0.849 | 1.047 | 1.299 | 1.676 | 2.009 | 2.109 | 2.403 | 2.678 | 2.937 | 3.261 | 3.496 |
| 60                             | 0.679                      | 0.848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 80                             | 0.678                      | 0.846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.088 | 2.374 | 2.639 | 2.887 | 3.195 | 3.416 |
| 100                            | 0.677                      | 0.845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.081 | 2.364 | 2.626 | 2.871 | 3.174 | 3.390 |
| 1000                           | 0.675                      | 0.842 | 1.037 | 1.282 | 1.646 | 1.962 | 2.056 | 2.330 | 2.581 | 2.813 | 3.098 | 3.300 |
| $z^*$                          | 0.674                      | 0.841 | 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.326 | 2.576 | 2.807 | 3.091 | 3.291 |
|                                | 50%                        | 60%   | 70%   | 80%   | 90%   | 95%   | 96%   | 98%   | 99%   | 99.5% | 99.8% | 99.9% |
|                                | Confidence level $C$       |       |       |       |       |       |       |       |       |       |       |       |